

# Effective Field Theory For Nuclei: Confronting Fundamental Questions in Astrophysics\*

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## Abstract

Fundamental issues involving nuclei in the celebrated solar neutrino problem are discussed in terms of an effective field theory adapted to nuclear few-body systems, with a focus on the proton fusion process and the *hep* process. Our strategy in addressing these questions is to combine chiral perturbation theory – an effective field theory of QCD – with an *accurate* nuclear physics approach to arrive at a *more effective* effective field theory that reveals *and* exploits a subtle role of the chiral-symmetry scale in short-distance effects encoded in short-range nuclear correlations. Our key argument is drawn from the close analogy of the principal weak matrix element figuring in the *hep* process to the suppressed matrix elements in the polarized neutron-proton capture at threshold currently being measured in the laboratories.

## 1 The Challenge

One of the currently exciting issues in astrophysics is the solar neutrino problem, which has recently been further highlighted by the Super-Kamiokande experiment [1]. Among the issues that have emerged from the recent measurement is the *hep* process  ${}^3\text{He}(p, e^+ \nu_e){}^4\text{He}$ . This reaction produces highest-energy solar neutrinos that may affect interpretations of the Super-K data in the astrophysical context and/or in search of evidence for new physics. The utmost importance of the *hep* gives nuclear physics a great challenge of providing reliable estimates of the *hep* cross sections. We would like to discuss this problem in this talk in light of the recent development in effective chiral field theories for nuclei that we (“PKMR”) have been developing for some time. Let us first state the problem and then develop the arguments that purport to meet this challenge.

### 1.1 The *hep* problem

The *hep* process,

$$p + {}^3\text{He} \rightarrow {}^4\text{He} + e^+ + \nu_e \quad (1)$$

in the Sun produces neutrinos of the maximum energy  $E_\nu^{\text{max}}(\text{hep}) = 18.795$  MeV, which is even higher than the maximum energy of the  ${}^8\text{B}$  neutrino,  $E_\nu^{\text{max}}({}^8\text{B}) = 17.980$  MeV. So the *hep* neutrinos near the upper end of the spectrum represent highest-energy solar neutrinos. However, the flux of the *hep* neutrinos is very small, because the Sun rarely uses this weak-interaction process to produce  ${}^4\text{He}$ ; the strong-interaction process,  ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2p$ , can produce  ${}^4\text{He}$  much more efficiently. In fact, the *hep* neutrino flux  $\phi(\text{hep})$  is even weaker than the  ${}^8\text{B}$  neutrino

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flux  $\phi(^8\text{B})$ , which is already much smaller than the primary  $pp\text{-}pep$  neutrino flux (see e.g., Fig. 2 of ref.[2]). On the other hand, since the  $hep$  reaction and  $^8\text{B}$  production occur at different stages of solar burning, the detection of the  $hep$  neutrinos is expected to provide information that is not obtainable from the  $^8\text{B}$  neutrinos. This provides a strong motivation for experimental efforts to measure the solar  $hep$  neutrinos. It is hoped that the fact  $E_\nu^{\text{max}}(hep) > E_\nu^{\text{max}}(^8\text{B})$  will allow identification of the  $hep$  neutrino despite its feeble flux which is almost drowned in the huge *background* of the  $^8\text{B}$  neutrinos.

This spectrum has been recently measured by the Super-Kamiokande collaboration (by measuring the energy spectrum of electrons recoiling from scattering with the solar neutrinos) [3]. The analysis of the electron energy spectrum [4] indicates that *if the observed spectrum were interpreted to be entirely due to the hep process*, then the fit would require – independently of neutrino oscillation scenarios – an enhancement in the  $hep$   $S$  factor by more than a factor of 20 relative to the Standard-Solar-Model (SSM) value  $S_0$ . This would imply that either the observed neutrinos are coming from some other sources than the  $hep$  or an important physics ingredient is missing that is signaling a new physics or else something is amiss in the strong interactions.

In this talk we shall address the last possibility, namely, the nuclear aspect of the problem. We shall thereby suggest how a reliable bound on the  $S$  factor can be given within the domain of strong interaction physics. The SSM incorporates the presently available “best” matrix element of the weak current [5], so the problem at hand boils down to asking whether the discrepancy is due to our inability to calculate the nuclear matrix element within a factor of as much as five.

Off-hand it would seem incredible that the highly successful standard nuclear physics approach could have been so wrong and for so long. The question is: Now that we know what the correct theory of strong interactions that must govern nuclear dynamics involved in the process is (i.e., QCD), why can’t nuclear theorists calculate reliably this matrix element and eliminate this big discrepancy? The frustration associated with this question is reflected in the recent remark by Bahcall [1]: “I do not see anyway at present to determine from experiment or from first principles theoretical calculations a relevant, robust upper limit to the  $hep$  production cross section (and therefore the  $hep$  solar neutrino flux).” This then makes “the range of values allowed by fundamental physics for the  $hep$  production cross section the most important unsolved problems in theoretical nuclear physics related to the solar neutrinos.”

The aim of this talk is to discuss how the above challenge could be met by means of exploiting effective field theories of QCD formulated for nuclear systems.

## 1.2 What makes this problem so tough: Chiral filter mechanism

As has been forcefully argued in this conference by Pandharipande [6] and Schiavilla [7], properties of light nuclei for mass number  $A \lesssim 9$  can be remarkably accurately calculated in a standard (highly sophisticated) potential-model approach with the potentials tuned to the wealth of available data. This approach, which we shall refer to as the “standard nuclear physics” approach, will later be incorporated in a scheme that is consistent with low-energy QCD. In measured electro-weak response functions – with the glaring exceptions that will be discussed below, this approach has proven to be stunningly accurate, so the question is what makes it so difficult to pin down the  $hep$  process matrix element better than a factor of four or more.

One way to see what goes hay-wire in the  $hep$  process is to use an old argument based on what is called “chiral filter conjecture” [8] which can be summarized as follows. Whenever leading single-particle processes receive contributions from many-body correction terms that involve one *soft-pion* exchange allowed by symmetry *and* unsuppressed by kinematics, then such terms – which are calculable accurately by means of low-energy theorems or current algebras – will dominate and higher-order corrections are both highly suppressed and systematically

calculable <sup>1</sup>. If on the other hand, one-soft-pion-exchange terms are absent due to symmetry or suppressed by kinematics, then higher-order corrections to the leading one-particle processes are not necessarily suppressed or convergent. This means that the calculation becomes highly model-dependent and that parameter-free predictions are not feasible. The former case will be referred to as “chiral-filter protected” and the latter as “chiral-filter unprotected.” The power of chiral filter is then to provide a simple rule of thumb as to which processes are easily calculable and which are not. However, even when processes in question are not protected by the chiral filter, it is in many cases still possible to carry out sufficiently accurate calculations. This is the case when the leading one-body terms have substantial contributions so that, even if next-order corrections cannot be assessed with precision, it does not cause a major problem. We will encounter such a situation in the proton fusion process in the Sun

$$p + p \rightarrow d + e^+ + \nu_e. \quad (2)$$

It turns out that this process can be calculated within an uncertainty of order of 8%. The reason for this is that the leading-order single-particle term dominates although higher-order corrections, unprotected by the chiral filter, could be uncertain by more than 100%. The same holds for the triton  $\beta$  decay.

The situation is drastically different in the cases of the *hep* process and of the suppressed isoscalar matrix elements in the polarized  $np$  capture  $\vec{n} + \vec{p} \rightarrow d + \gamma$ . For the process (1) which undergoes primarily through an axial weak current, the lepton momentum transfer is small, so one would, off-hand, think that the leading term would be the allowed Gamow-Teller (GT) matrix element. But since the GT operator just flips spin and isospin, the matrix element involving the initial and final hadronic states in (1) is suppressed by the orthogonality of the wave functions involving different spatial symmetries in the main components of the wave functions: in the Young tableaux notation of the symmetry group  $S_4$ , the initial state is in [31] and the final state in [4]. With realistic wave functions, the matrix element is not quite zero but tiny [5, 7]. This means that for a reliable calculation of the process, we need to have the correction terms under a quantitative and systematic control. The argument of [8], however, says that they are unprotected by the chiral filter and hence cannot be controlled in a straightforward way. As we will see later, the situation is further exacerbated by the fact that the correction terms come with an opposite sign to the “leading” matrix element, causing a serious cancellation.

So how do we go about computing this process with any confidence at all? This is the Bahcall challenge. We propose that a tool to meet this challenge is to combine effective field theory with the standard nuclear approach as presented by [6, 7] and to formulate a *hybrid approach*, which we shall call “*more effective* effective field theory (MEEFT).”

## 2 Effective Field Theory for Nuclei

### 2.1 Effective field theory (EFT) defined

The only known way to answer low-energy nuclear physics questions from fundamental principles is to resort to effective field theory adapted to nuclear few-body problems. At low energy, the relevant degrees of freedom are not the confined quarks and gluons of QCD but non-strange baryons and mesons. Doing a QCD calculation in nuclear physics translates into doing an effective field theory, which done *fully* is no more and no less than QCD proper [12]. So what is an effective field theory for nuclei which we may qualify as “fundamental”?

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<sup>1</sup>The well-tested processes belonging to this class are the unpolarized cross section for thermal neutron capture by proton  $n + p \rightarrow d + \gamma$  [9], electrodisintegration of the deuteron  $e + d \rightarrow n + p + e$  [10] (both of which are dominated by the isovector M1 operator) and weak axial charge nuclear transitions  $A(J^\pm \rightarrow J^\mp)$  [11].

Phrased in a nut-shell and somewhat loosely, it goes as follows: Suppose that we are interested in a process whose energy (momentum) scale is much less than, say,  $\Lambda$ . Consider a generic field  $\Phi$  that accounts for the physics we are interested in. Let the degrees of freedom lodged above the scale  $\Lambda$  – in which we *are not* interested – be denoted by  $\Phi_H$  and those lodged below  $\Lambda$  – in which we *are* interested – by  $\Phi_L$ . The physics is in the “partition function”  $\int [d\Phi] \exp(iS(\Phi))$ . Since we are not interested in  $\Phi_H$  we may integrate it out and express the partition function in terms of an effective action  $S^{eff}(\Phi_L)$  as  $\int [d\Phi_L] \exp(iS^{eff}(\Phi_L))$  where  $\exp(iS^{eff}) = \int [d\Phi_H] \exp(iS(\Phi_H, \Phi_L))$ . This is just a formal manipulation, so nothing will be gained unless one can arrive at a manageable  $S^{eff}$  – expressed in terms of a reduced number of relevant degrees of freedom that we can control – with which we could do physics systematically. The next step is to approximate the effective action, in general *non-local*, by an infinite sum of *local* terms as

$$S^{eff}(\Phi_L) = \sum C_n \mathcal{O}_n \quad (3)$$

where  $\mathcal{O}$ ’s are local operators involving the field  $\Phi_L$  and  $C$ ’s are coefficients dependent on the scale  $\Lambda$  to be determined either theoretically from first principles or from experiments. Since physical quantities should not depend upon where one puts the separation or cutoff  $\Lambda$ , the coefficients  $C_n$  should satisfy the renormalization-group (RG) flow equation (known as the Wilson RG equation)

$$\partial C_n(\Lambda) / \partial \Lambda = \mathcal{F}(\Lambda). \quad (4)$$

If the theory is completely defined everywhere, that is, both above and below  $\Lambda$ , then one may do the integrating-out explicitly and obtain an effective action. By a procedure of “matching,” one could fix  $C_n$  term by term. In nuclear physics, this is not possible since the relevant degrees of freedom are not the same in the quark-gluon and hadron regimes and there is no known way to go from one to the other. In this case the best one can do is to truncate the series at low enough order and resort to a certain number of “clever guesses.” It is at this stage that one needs to have a guiding principle for making right guesses and reliable error estimates. This necessarily injects a certain degree of “art” in the actual calculation.

If the right-hand side of eq.(4) goes to zero at some  $\Lambda$ , then it is a signal that there is a fixed point. This is the case for Landau Fermi liquid parameters [13] in many-body systems. An effective field theory for nuclear matter that exploits the Fermi-liquid fixed points and BR scaling [14] has been proposed in [15]. In the problem at hand, this aspect plays no role, so we will not discuss it any further.

## 2.2 More effective EFT

In dealing with electro-weak response functions and more significantly with  $n$ -body systems with  $n > 1$ , we find the hybrid approach developed over the years by PKMR [16] to be more predictive than its cousins [17] and in some cases to be the *only* predictive method, although for two-body systems, they are more or less equivalent. The hybrid approach – which follows Weinberg’s proposal [18, 19] – consists of distinguishing two sets of graphs, one “irreducible graphs” and the other “reducible graphs.” In treating the irreducible graphs that involve no infrared enhancements, one uses standard chiral perturbation theory. This generically involves an electro-weak vertex  $\Gamma$  (which enters at most once as we will be dealing with a slowly varying external field) and a two- or many-body interaction potential  $V$  computed to a finite order of chiral expansion. The reducible graphs involving  $V$  and infrared-enhanced propagators on the other hand need to be iterated to all orders, a procedure which is effectively executed in Lippman-Schwinger equation or Schrödinger equation. The hybrid strategy is then to take the

accurate wave functions generated by the procedure as described by Pandharipande and Schiavilla in this meeting and compute the relevant matrix elements with the current vertex computed in chiral perturbation theory to as high an order as feasible which in practice turns out to be next-to-next-to leading order. Beyond that order, the theory becomes unproductive due to the paucity of experimental data.

We should mention a caveat to this (hybrid) procedure: Since the reducible graphs are iterated to all orders while the irreducible graphs (i.e., current vertex  $\Gamma$  and potential  $V$ ) are computed to a finite order  $n$ , one makes an error at order  $n+1$  in the EFT counting. This caveat can be avoided in principle as e.g. in the approach advocated by the authors in [17]. However in practice, the counting error at the  $(n+1)$  order makes a negligible numerical error in all chiral-filter-protected observables. In fact one can show rather convincingly that in two-body systems at low energy, the two methods are essentially equivalent [20].

We propose that this hybrid theory is *more effective* than an effective theory of the type advocated in [17] for certain processes. The ingenuity is to realize that one can turn the above caveat around to advantage when one is calculating three- or more-body systems and, perhaps more significantly, when the chiral-filter protection is simply missing as in the *hep* and *np* processes.

### 2.3 Power counting

Computing the irreducible current vertex  $\Gamma$  and the potential  $V$  is systematically organized by a power counting rule which in the present case is the usual chiral counting [18]. Let the characteristic probe energy/momentum scale be given by  $Q$  and the cutoff that separates the “high (H)” and “low (L)” regimes be  $\Lambda$ . We are interested in making an expansion in  $Q/\Lambda$  with a series  $C_\nu(Q/\Lambda)^\nu$ . In *more effective* EFT (MEEFT), the potential is taken from nature – a procedure which is consistent with the tenet of effective field theories contrary to misunderstandings among some theorists, so it suffices for our purpose to focus solely on the current vertex. The index  $\nu$  for an  $n_B$ -body current is given by [18]

$$\nu = 2(n_B - 1) + 2L + \sum_i \nu_i \quad (5)$$

where

$$\nu_i = d_i + e_i + n_i/2 - 2 \quad (6)$$

and where  $L$  is the number of loops,  $n_i$  the number of nucleon lines entering the  $i$ th vertex,  $d_i$  the number of derivatives and  $e_i$  the number of external fields. We are dealing with a slowly varying field so the external field acts only once, i.e.,  $e_i = 1$ .

In discussing higher order corrections, we shall denote by  $N^\nu\text{LO}$  the  $\nu$ -th order correction *relative* to the LO.

It is worth noting at this point that relative to one-body currents, the leading 2-body currents are  $N^1\text{LO}$  if protected by the chiral filter and  $N^3\text{LO}$  if not. This is because the sum  $\sum_i \nu_i$  is different by 1 between the LO 1-body and LO 2-body currents.<sup>2</sup>

### 2.4 Chiral constraint

Chiral symmetry has an important constraint on  $\nu_i$ , namely that  $\nu_i \geq 0$ . The presence of an external field in this constraint has a non-trivial consequence on the chiral filter as pointed out in [21]. This is because power counting is subtle when an external field figures. In the Weinberg

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<sup>2</sup>For example, if we look at  $\sum_i \nu_i$  for the (LO 1-body, LO 2-body) contributions, we find (1, 0) for the isovector M1 and the axial charge, (1, 2) for the isoscalar M1, and (0, 1) for the GT.

counting [18] which we adopt here and in the absence of external fields (that is, in the potential), the leading one-pion exchange and leading four-Fermi contact interaction terms are of the same chiral order, so the long-range one-pion exchange and the contact (short-range) interaction are equally important. However when an external field is present, the contact interaction becomes sub-dominant to one-pion exchange [21]. This means that short-range many-body currents are of higher order in chiral counting than a one-pion-exchange current. This obvious point turns out to be essential for understanding the chiral filter mechanism. A consequence is that if one were to calculate  $n$ -body currents by insisting that they be strictly consistent with the corresponding potential (e.g., by imposing current conservation), then to be consistent *also* with chiral symmetry, other terms of the same chiral order as the short-ranged current – such as *pion loops and counter terms* – must be included. More specifically, this implies that the procedure often used in the literature to calculate two-body currents with the exchange of all the mesons that figure in the one-boson-exchange potential model, namely,  $\pi$ ,  $\rho$ ,  $\omega$ ,  $\sigma$  etc. is incomplete and hence could be erroneous numerically unless supplemented with additional counter terms and loop terms of the same chiral orders.

## 2.5 Two scales

There are basically two relevant scales in the problem: “nuclear scale” and “chiral scale.” In the low-energy regime we are concerned with, the nuclear scale is given by one or two pion masses depending upon whether one introduces the pion explicitly or integrates it out. A viable theory must be stable around this nuclear scale. It has been verified that this stability is easily assured by all consistent EFTs [16, 17].

The chiral scale is essentially given by spontaneously broken chiral symmetry characterized by the scale  $\sim 4\pi m_\pi \sim m_N \sim 1$  GeV. This scale delineates vector mesons  $\rho$ ,  $\omega$ , glueballs etc. from the low-energy regime and short-distance physics from long-distance physics. In our treatment, nucleons and pions are introduced explicitly while all others are integrated out, so the chiral scale basically corresponds to the scale at which short-range correlation of the standard nuclear physics approach enters, namely at the scale of vector meson masses.

## 3 Polarized Neutron-Proton Capture: Hint for a Strategy

### 3.1 Hard-core cutoff scheme (HCCS)

The capture cross section for  $n + p \rightarrow d + \gamma$  is dominated by an isovector  $M1$  operator (denoted  $M1V$ ) which is chiral-filter protected according to [8, 21] and hence is very accurately predicted [9]. Meanwhile, the matrix elements of the isoscalar electromagnetic current are suppressed by a factor of  $\sim 10^3$ . Thus they cannot be “seen” in the total cross section. However the isoscalar  $M1$  (called  $M1S$  to distinguish it from the isovector  $M1$ ) and  $E2S$  (isoscalar  $E2$ ) can be extracted via spin-dependent observables from the polarized capture process

$$\vec{n} + \vec{p} \rightarrow d + \gamma \tag{7}$$

which is being measured at Institut Laue-Langevin (ILL) in Grenoble [22].

We will now make predictions for these matrix elements [23]. In doing so, we will learn how to compute the *hep* process. There is a very close parallel between the two.

Very much like in the *hep* process, the  $M1S$  matrix element is highly suppressed due to the orthogonality between the initial wave function  $^3S_1$  and the final deuteron wave function. This makes the  $M1S$  as suppressed as  $E2S$ . Furthermore the isoscalar EM current is *not* protected by the chiral-filter mechanism, so the corrections can be uncomfortably large and non-controllable.

There is a way out of this impasse and it is found in the exploitation of the two ingredients of the MEEFT [23]. Firstly the initial and final wave functions are very accurately known [24] and secondly to next-to-next-to leading order (N<sup>4</sup>LO) (where the leading order (LO) corresponds to the single-particle  $M1S$  matrix element), the loops and counter terms required by chiral symmetry are controlled by the chiral cutoff  $\Lambda$  reflecting short-distance physics involving a scale  $\Lambda \sim 4\pi f_\pi \sim m_N \sim 1$  GeV. There are four independent counter terms that arise at N<sup>3</sup>LO and N<sup>4</sup>LO. Most remarkably, *to the order we are working in, they combine to a single constant  $X$  in the precise form that is required to fix completely the deuteron magnetic moment for a given  $\Lambda$ .* We stress that, were it not for this “coincidence,” there would be too many undetermined parameters and it would be impossible to make a prediction for  $M1S$ . It should also be mentioned that this unique correspondence is probably possible only to N<sup>4</sup>LO. We suspect that to higher orders, the one-to-one correspondence that makes the prediction possible would be spoiled.

Now the tenet of a viable EFT is that the result should not depend on the cutoff  $\Lambda$  as it represents the scale at which  $\Phi_H$  and  $\Phi_L$  are separated – which is arbitrary. The degrees of freedom that have been integrated out in [23] are heavy mesons (such as  $\rho$ ,  $\omega$ , glueballs etc.). Any strong dependence on  $\Lambda$  would therefore signal that certain short-distance physics is missed and/or that we have a non-converging series. *What we have done in fixing the counter terms at a given  $\Lambda$  to the empirical magnetic moment of the deuteron corresponds to a specific regularization scheme called “hard-core cutoff scheme (HCCS)”*<sup>3</sup> which subsumes certain terms higher order than N<sup>4</sup>LO as required by the empirical deuteron magnetic moment. In principle, the counter terms are *presumably* calculable (in the far future) order by order from “first principles” for a given  $\Lambda$ . However since we are using a particular regularization scheme, the counter terms we fix from experiments are *not* necessarily calculable order by order.

### 3.2 Prediction

What the experimenters are measuring are the ratios  $\mathcal{R}$ , i.e., the  $M1S$  and  $E2S$  relative to the isovector M1 ( $M1V$ ),

$$\mathcal{R}_{M1S} = \frac{M1S}{M1V}, \quad \mathcal{R}_{E2S} = \frac{E2S}{M1V}. \quad (8)$$

In [23], regularization was effected with the distance cutoff  $r_c$  instead of a momentum cutoff. This cutoff is closely related to the “hard-core radius” used in the standard nuclear physics approach. The range of  $r_c$  we have explored was

$$r_c^{min} \equiv 0.01 \text{ fm} \leq r_c \leq r_c^{max} \equiv 0.8 \text{ fm}. \quad (9)$$

This is wide enough to amply cover the range implied by spontaneously broken chiral symmetry. Expressed in the chiral order and in the range  $(r_c^{max}, r_c^{min})$ , the  $M1S$  comes out to be

$$\begin{aligned} \mathcal{R}_{M1S} \times 10^3 &= \text{“LO”} + \text{“N}^3\text{LO”} + \text{“N}^4\text{LO”} + \text{“(N}^3\text{LO} + \text{N}^4\text{LO)}_X\text{”} \\ &= -0.74 + (-0.48, -0.74) + (0.23, 0.46) + (0.49, 0.54) \\ &= (-0.50, -0.49) \end{aligned} \quad (10)$$

where the contribution that depends on the single parameter  $X$  fixed by the deuteron magnetic moment is indicated by the subscript  $X$ . The crucial observation here as well as later in the case of the *hep* process is that the final result cannot be arrived at by any partial sum of the terms. Note also the important role of the  $X$ -dependent term.

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<sup>3</sup>In [23], this scheme was referred to as “MHCCS.”

The  $E2S$  on the other hand does not depend on  $X$  and the correction from higher orders ( $N^3LO$  and  $N^4LO$ ) to the LO result is negligible

$$\mathcal{R}_{E2S} \times 10^3 = 0.24 + \mathcal{O}(10^{-3}). \quad (11)$$

The lessons we learn from this prediction are two-fold. One, because of the strong suppression of the LO term, the corrections are all comparable independently of the chiral order and not small compared to the LO. Second, the final result is practically independent of the cutoff  $r_c$ , assuring that no important part of short-distance physics – presumably encoded in short-range correlation – is missed in the procedure. It suggests that the regularization procedure adopted here in some sense *forces* the series to converge at  $N^4LO$ .<sup>4</sup>

## 4 Solar Neutrino Processes

### 4.1 Chiral filter protection/non-protection

Since the lepton momentum transfer in the process (1) – which is mediated by the isovector axial current  $A_\mu^a$  – is small, the leading-order (LO) term in the current is the one-body Gamow-Teller (GT) operator with index  $\nu = 0$  coming from the space component  $\vec{A}^a$ . The next-order terms are first-forbidden corrections to the one-body GT operator and the axial charge one-body contribution. In the results reported here, the former will be incorporated into the LO term. The latter will be treated separately.

The axial-charge operator  $A_0^a$  is *protected* by chiral filter [8]: corrections to the one-body axial charge operator (of order  $\nu = 2$  and  $N^1LO$ ) are dominated by one-soft-pion exchange. Thus corrections to order  $\nu = 2$  are completely fixed. Further corrections to the axial charge are strongly suppressed and can be ignored [11].

On the contrary, the space component  $\vec{A}^a$  in which the one-body GT operator is the “leading” term is *not* protected and so the  $n$ -body corrections are generically uncontrollable. It is this part that challenges the theorists. Most fortunately, however, it turns out that the procedure exploited for the neutron-proton capture process applies in a complete parallel to the *hep* process.

### 4.2 The proton fusion

Before we tackle the *hep* problem, consider the proton fusion process (2) which is predominantly given by the unsuppressed one-body Gamow-Teller (GT) operator. For this, the axial-charge operator does not figure. Since the space component of the axial current is not protected by the chiral filter, corrections to the GT term – that are two-body – can be calculated only to  $N^3LO$  which involve no loops and no counter terms. To  $N^3LO$ , one obtains [27]

$$M_{GT} = 4.77(1 + 0.04). \quad (12)$$

Although the  $N^3LO$  correction (given in the parenthesis in (12)) is only 4%, there is nothing that would suggest that the  $N^4LO$  and higher order terms can be ignored compared to the  $N^3LO$ . Unfortunately to  $N^4LO$ , there are too many unknown counter terms to make an unambiguous estimate. The error could well be 100% or more of the  $N^3LO$  term.

Even so, given that the one-body GT term is dominant, it still makes sense to compute the solar  $S$  factor using (12)<sup>5</sup>,

$$S_{pp}^{th} = 4.05(1 \pm 0.08) \times 10^{-25} \text{MeVb} \quad (13)$$

<sup>4</sup>It is quite remarkable that the same results were obtained in a different procedure by Chen, Rupak and Savage [25], i.e.,  $\mathcal{R}_{M1S} = -0.50(1 \pm 0.6)$  and  $\mathcal{R}_{E2S} = 0.25(1 \pm 0.15)$ .

<sup>5</sup>The “error” quoted here represents a range of uncertainty involved at  $N^4LO$  which is not presently calculable in a parameter-free way and should not be taken quantitatively.



which is close to the value adopted in the standard solar model [26]

$$S_{pp}^{SSM} = 4.00(1 \pm 0.007_{-0.011}^{+0.020}) \times 10^{-25} \text{MeVb}. \quad (14)$$

This result illustrates a case where the leading term is unsuppressed and corrections to it are sufficiently small so that the chiral filter non-protection does not spoil the predictive power of the calculation although greater accuracy cannot be achieved without further experimental inputs that are not expected to be forthcoming in the near future. A recent calculation [28] using the approach of [17] supports the prediction of [27] although because of one free parameter, the result of [28] is not, strictly speaking, a genuine prediction.

We note without any details that a similar situation holds for the triton beta decay. There are four parameters at N<sup>3</sup>LO and N<sup>4</sup>LO that cannot be determined at present either by theory or by experiments. It turns out however that they can be combined into one parameter – denoted  $Y$  – that enters into the  $hep$  matrix element.

### 4.3 The $hep$ process

As mentioned the axial-charge contribution is unambiguously calculable, so we will not go into it: We see no obstacle to calculating it with accuracy. We will therefore focus on the main quantity at issue, the GT term and  $n$ -body corrections thereof. A precise calculation of these quantities is possible because, as in the case of  $M1S$  in the neutron-proton capture, the counter terms – there are four of them – at the N<sup>3</sup>LO and N<sup>4</sup>LO order can be combined precisely into a single constant  $Y$  that supplies the only undetermined constant present at N<sup>4</sup>LO in the triton beta decay  ${}^3H \rightarrow {}^3He + e^- + \nu_e$ . By fixing the constant  $Y$  from the triton beta decay, the theory for the controversial term in the  $hep$  process is completely determined. We have verified this only up to N<sup>4</sup>LO, beyond which, we suspect, such a “miracle” may cease to be operative. Expressed in the ascending chiral order and in the momentum cutoff range ( $\Lambda_{min} = 0.5\text{GeV}$ ,  $\Lambda_{max} = 1.0\text{GeV}$ ), we find with the Av14 wave function <sup>6</sup>

$$\begin{aligned} M_{GT}/g_A &= \text{“LO”} + \text{“N}^3\text{LO”} + \text{“N}^4\text{LO”} + \text{“(N}^3\text{LO} + \text{N}^4\text{LO)}_Y\text{”} + \text{“N}^4\text{LO}_{3body}\text{”} \\ &= -0.38 + (0.69, 1.50) + (0.24, -0.94) + (-0.74, -0.43) + (0.002, 0.004) \\ &= (-0.19, -0.24). \end{aligned} \quad (15)$$

Note that as mentioned, the LO and N<sup>3</sup>LO terms come with opposite signs resulting in the cancellation mentioned before. Although negligible, three-body currents that appear at N<sup>4</sup>LO are included for comparison. There are no loop contributions at this order and since only pion exchanges are involved, no unknown constants are involved. It should be noticed that as in the case of the suppressed isoscalar matrix elements in the  $np$  capture, the  $Y$ -dependent contribution is crucial.

Our final value with an error estimate is [29]

$$M_{GT}/g_A = -0.20 \pm 0.06. \quad (16)$$

As in the  $np$  case, due to the strong suppression of the leading term, all contributions come in with more or less equal strengths. Furthermore as in the case of the polarized  $np$  capture, no partial sum of the terms resembles the final result. It is only in the total that the strong  $\Lambda$  dependence disappears. Like in the polarized  $np$  case, we interpret the approximate  $\Lambda$ -independence (for the wide range of  $\Lambda$  considered) as an indication that short-distance physics is properly captured in the regularization scheme.

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<sup>6</sup>One can perhaps do better with the Av18 wave functions as in [7] but we believe what we used is good enough for the present purpose.

## 5 Implication On The *hep* Problem

In this note, we have suggested how to pin down the principal matrix element governing the process (1) that has frustrated for years the effort to calculate a reliable bound for the *hep*  $S$  factor. Once we obtain the chiral-filtered and hence accurately calculable contribution from the axial charge operator, we will be in a position to confront directly the Super-Kamiokande data from the point of view of strong-interaction physics.

While awaiting the numerical value of the axial-charge matrix element as well as the re-confirmation of (16) (in progress at the time of this writing), we can still make a reasonable qualitative statement. Since the axial-charge contribution is of the same order as the first-forbidden term, it cannot be much greater than the GT term. Tenuous though it might be, some support for this conjecture comes from the recent result of Marcucci et al [7] who estimated the axial-charge two-body currents to contribute about 40% to the  $S$  factor. It should however be cautioned that since their approach to the axial-charge operators is different from ours, we cannot carry it over directly to our result. Be that as it may, for the sake of argument, we shall assign a generous error of 100% to the matrix element that we computed to account for the axial-charge contribution and possible first-forbidden corrections that still need to be accounted for. This would give the ratio of the predicted  $S$  factor  $S^{th}$  to the Super-Kamiokande observation  $S^{SK}$

$$\frac{S^{th}}{S^{SK}} \lesssim 0.10 \sim 0.12 \quad (17)$$

more or less independently of the neutrino oscillation scenarios [1] (i.e., MSW or vacuum). There is still an order of magnitude “missing” in the  $S$  factor which cannot be accounted for by strong interaction mechanisms only.

The conclusion then is either (1) the Super-Kamiokande experiment is wrong, (2) the experiment is correct but the interpretation of the data is incorrect (e.g., the observed events could be due to “backgrounds” such as supernovae) or (3) there is new physics in the sectors outside of strong interactions. Which of these alternatives is the viable one could probably be settled by such laboratory experiments [30] as  $n + {}^3\text{He} \rightarrow {}^4\text{He} + e^- + \bar{\nu}_e$  and  $e^- + {}^4\text{He} \rightarrow {}^3\text{H} + n + \nu_e$ .

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